Categories for the beginning mathematicians

Grad student: Raymond Chan

Description: Category theory has been a fundamental language throughout modern mathematics, and this seminar aims to provide early exposure to relatively young mathematicians.

Every kind of mathematical object comes with the notion of "appropriate maps" between them. For sets, we have functions; for topological spaces, we have continuous functions; for vector spaces, we have linear maps; for groups, we have group homomorphisms ... It turns out the framework of set theory is not ideal for capturing all such data at once (objects + maps), since a set is determined by its elements, and the only possible relationship between elements is equality. (Technically you can, and some people choose to do so to avoid logical paradoxes, but our treatment will completely ignore such technicality, making it much more accessible to nonexperts.)

The way to resolve this issue is a rather genius (or stupid) trick: you just invent a thing that allows you to put all these data together, and you get a category. (That's not how categories were born historically, but whatever.) However, by elevating things to the categorical level, we lose access to the details of each individual object! So from the categorical perspective, we no longer have the lotion of elements even though the objects in question might be set-based. But this also means we can use categorical language even when the objects in question are not set-based, and we will see some examples of such categories. Miraculously, we can still distinguish the objects from one another by looking at their relationships as a whole. We will explore this a little bit during the first meeting on the level of sets.

Expectation from students: Students will sign up for presentations (which is *not* a strict requirement). We aim to have at most 2 students presenting each week. The presentations are *not* meant to demonstrate how much the presenters have mastered the material, and hence presenters should bring any questions onto the table and discuss them with the audience. The audience should also raise any questions they have during the presentations as well, and the presenters will answer them. Since examples of the reading will be drawn from different areas of mathematics, students will support each other by filling in the gaps for those who are not familiar with a particular concept.

Prerequisite: Familiarity with sets and functions will be assumed (at the level of MAT 108). Familiarity with other mathematical objects (topological spaces, groups, rings, etc.) is useful but not strictly required.

Reading: There are two main references, but we will mainly stick with the first one:

(1) Tom Leinster, "Basic Category Theory" (arXiv:1612.09375), Chapter 0,1,2,4,5.

(2) Emily Riehl, "Category Theory in Context" (available for free online)

Exercises will be assigned along with the reading, and the students should try some of them each week. Some will be easy, some will be hard.

Outline See below for the (rather ambitious) schedule, subject to some minor changes depending on progress.

Meeting 1	Get to know each other The unimportance of elements, commutative diagrams, (maybe) universal property [Chapter 0]
Meeting 2-3	Categories, examples, and some concepts living inside categories [Chapter 1.1 + monics and epics from Chapter 5] Functors and examples [Chapter 1.2] Natural transformations, examples, equivalence of categories [Chapter 1.3]
Meeting 4-5	Representable functors [Chapter 4.1] Adjoint functors [Chapter 2.1, 2.2]
Meeting 5-6	Yoneda Lemma, Yoneda embedding, and universal elements [Chapter 4.2, 4.3]
Meeting 7(-8)	Limits and colimits [Chapter 5.1, 5.2]
Meeting 8	A choice of further topic (monoidal category, monad, abelian categories, Chapter 5.3/6 of the book, etc.)
Meeting X	Catch up week, skip for whatever reason, or a choice of further topic